Superconducting orbital diode effect in SN bilayers

II international conference «Superconductivity in Nanostructures», September 21-27, 2025

Yuriy Dmitrievtsev[1]*, Ya. V. Fominov[1,2]

[1]L. D. Landau Institute for Theoretical Physics RAS
[2]Moscow Institute of Physics and Technology

*dmitrievtsev.iua@phystech.edu

The orbital diode effect in the SN bilayer (superconductor - normal metal) is investigated.

- Dirty limit: $l \ll \xi, d$
- In-plane **magnetic field B** → Violation of time reversal symmetry..
- **Inhomogeneity** along the thickness → Gradient of density of superconducting electrons $\nabla \mathbf{n}$ → Violation of geometric symmetry.
- Particular direction $\nabla \mathbf{n} \times \mathbf{B}$ + irreversibility in time → *Diode effect* $I_{c+} \neq I_{c-}$

The effect was predicted by M. Yu. Levichev, I. Yu. Pashenkin, N. S. Gusev and D. Yu. Vodolazov in the article [1], where it has been studied numerically and experimentally.

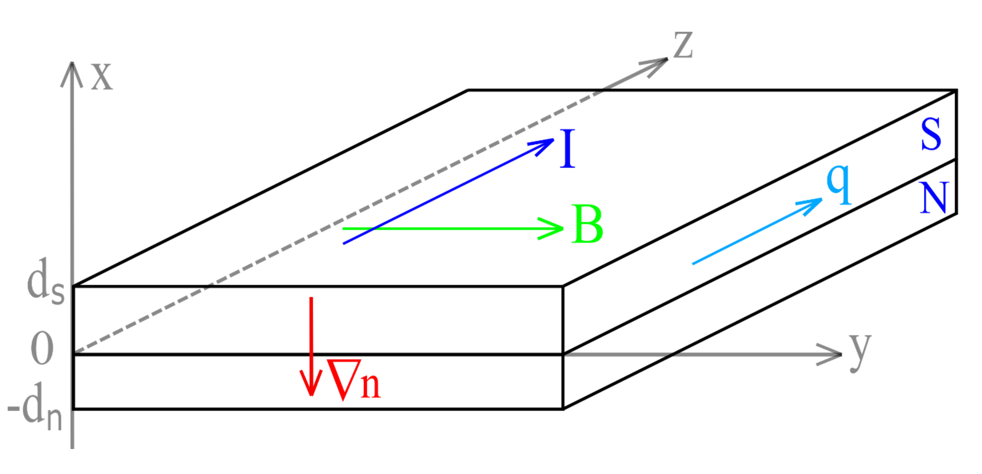

Fig. 1. SN bilayer

Our goal – *analytical description + explore the influence of the interface*.

Analytical solution is possible only in the weakly inhomogeneous case:

1. Thin bilayer $d \ll \xi$ + arbitrary transparency of the interface (similar to [2])
2. Thick bilayer $d \geq \xi$ + low-transparent interface

There is no effect in the zero order of inhomogeneity.

The key role is played by **inhomogeneities**

- The homogeneity condition:

$$\frac{d^2}{\xi^2} \ll max\left(1, \frac{R}{R_0}\right) \qquad R_0 = R_q \frac{\delta_S + \delta_N}{|\Delta|}$$

Here:
R - Interface resistance
R_q - Quantum resistance
$\delta_{S,N} = (\nu_{S,N} V_{S,N})^{-1}$ - level spacing

- Supercurrent as a function of momentum and magnetic field:

$$\frac{I(q,B)}{I_0} = \frac{n_{eff}(q,B)}{n_0}\frac{q}{q_c} + k\left|\frac{\Delta(q,B)}{\Delta_0}\right|^2 \frac{B}{B_c}$$

Here:
n_0, Δ_0 - density and order parameter at zero temperature $T = 0$ in the absence of a magnetic field $B = 0$
q_c, B_c - critical momentum and magnetic field
I_0 - characteristic current value (current at $n_{eff} = n_0$, $q = q_c, B = 0$)
k - small dimensionless parameter, depending on the parameters of the bilayer and the resistance (transparency) of the interface
Parameter k is small due to the smallness of inhomogeneities

There are 2 sources of **inhomogeneity**:

1. «Inherent» inhomogeneity:

   Is related to the finite thickness of the bilayer compared to the coherence length.

   Depends on the thickness d/ξ .

2. Inhomogeneity, created by the interface:

   Related to the finite transparency of the interface.

   Depends on the resistance (transparency) of the interface R/R_0 .

Supercurrent as a function of momentum of superconducting electrons and magnetic field

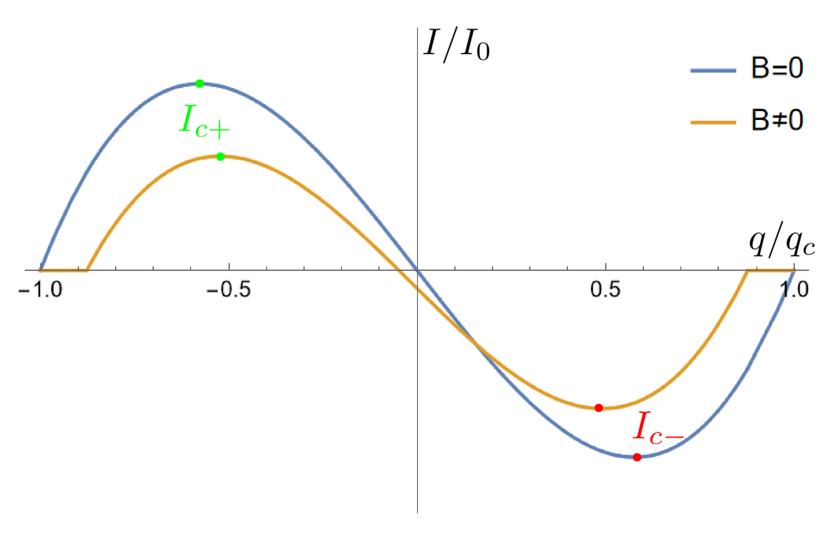

Fig. 2. The dependence of current on momentum in the absence/presence of a magnetic field.

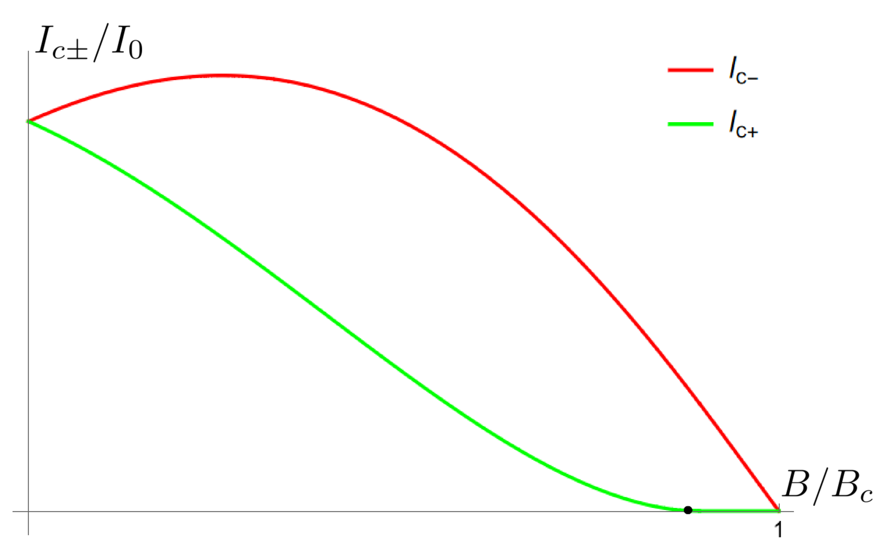

Fig. 3. Dependence of critical currents flowing in opposite directions on the magnetic field.

Dependence of the **strength of the diode effect** on the transparency (resistance) of the interface

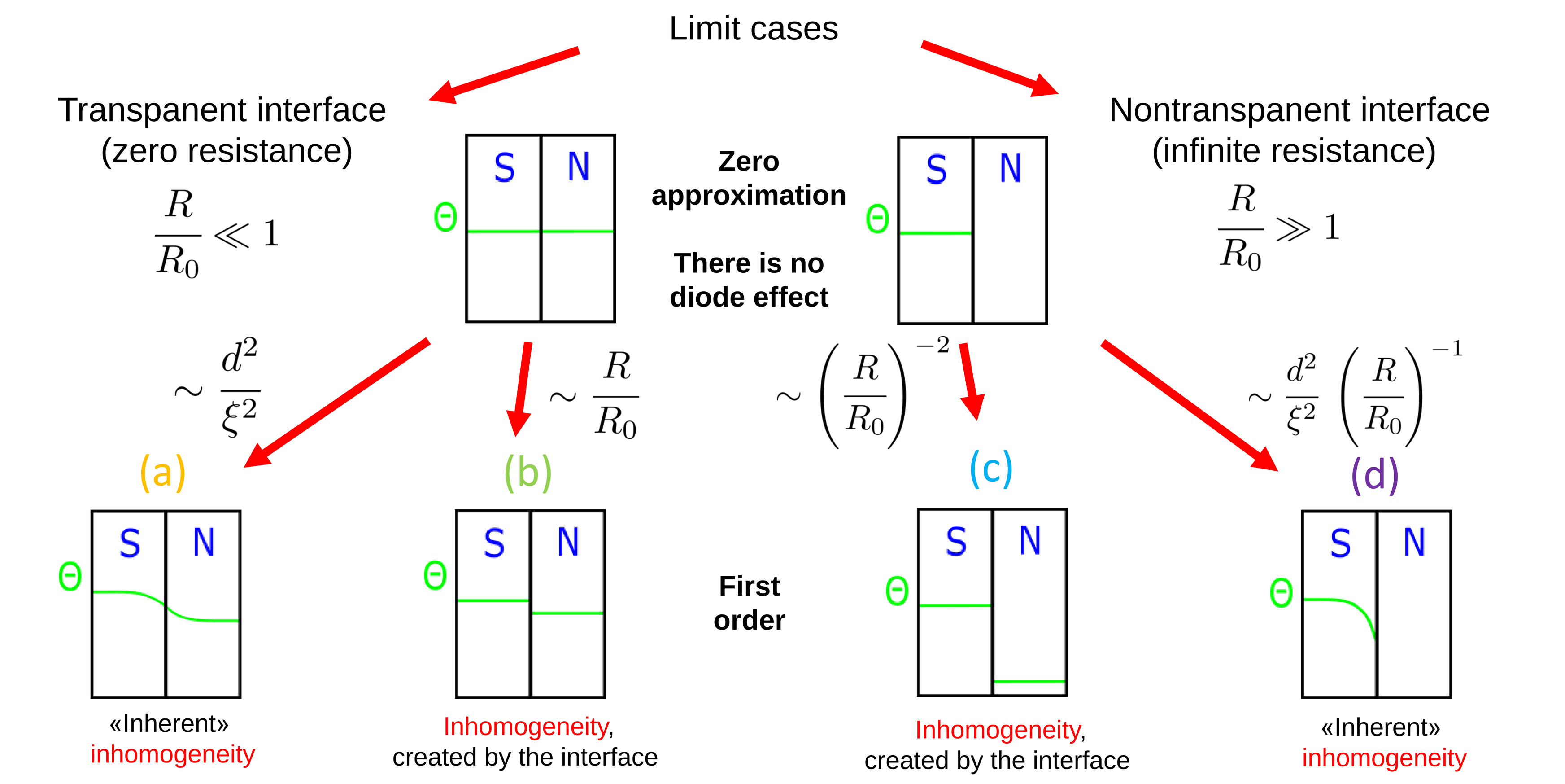

Fig. 6. Inhomogeneities in the case of low resistance (transparent interface).

Fig. 7. Inhomogeneities in case of high resistance (nontransparent interface).

Strength of the diode effect

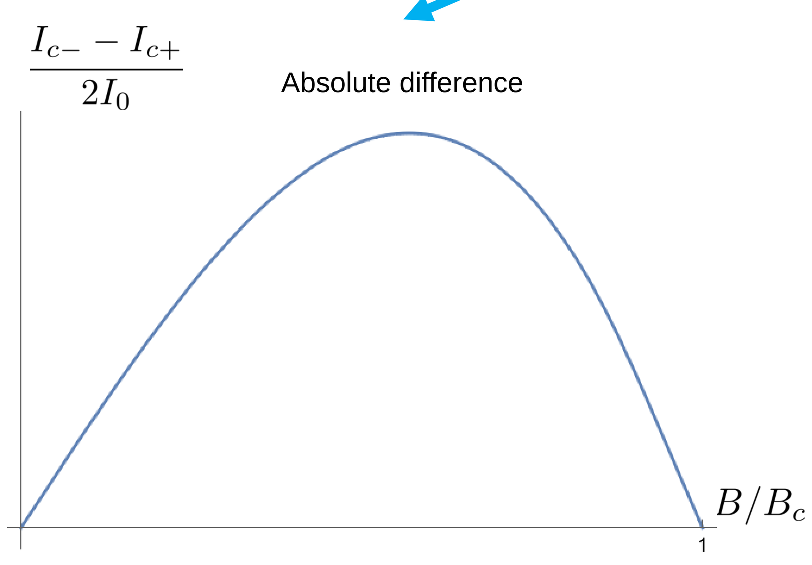

Fig. 4. Dependence of the absolute difference of critical currents on the magnetic field.

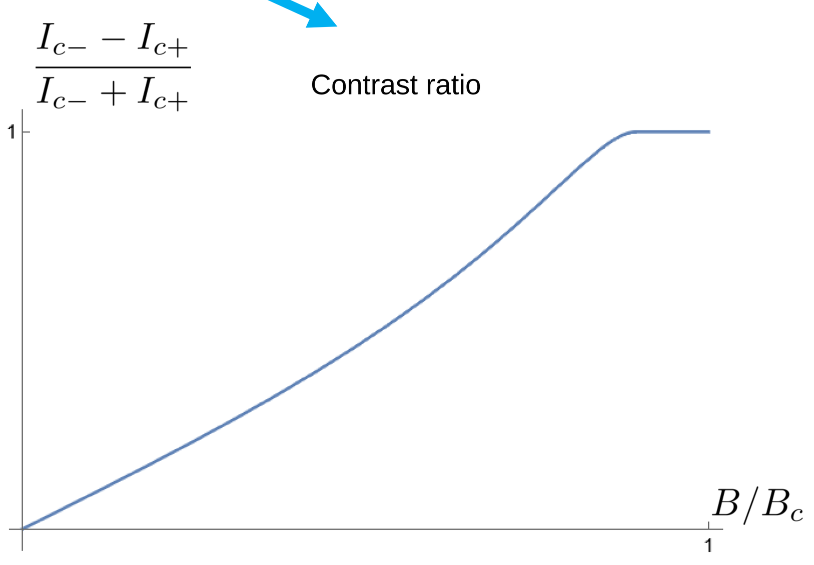

Fig. 5. Dependence of contrast ratio on the magnetic field.

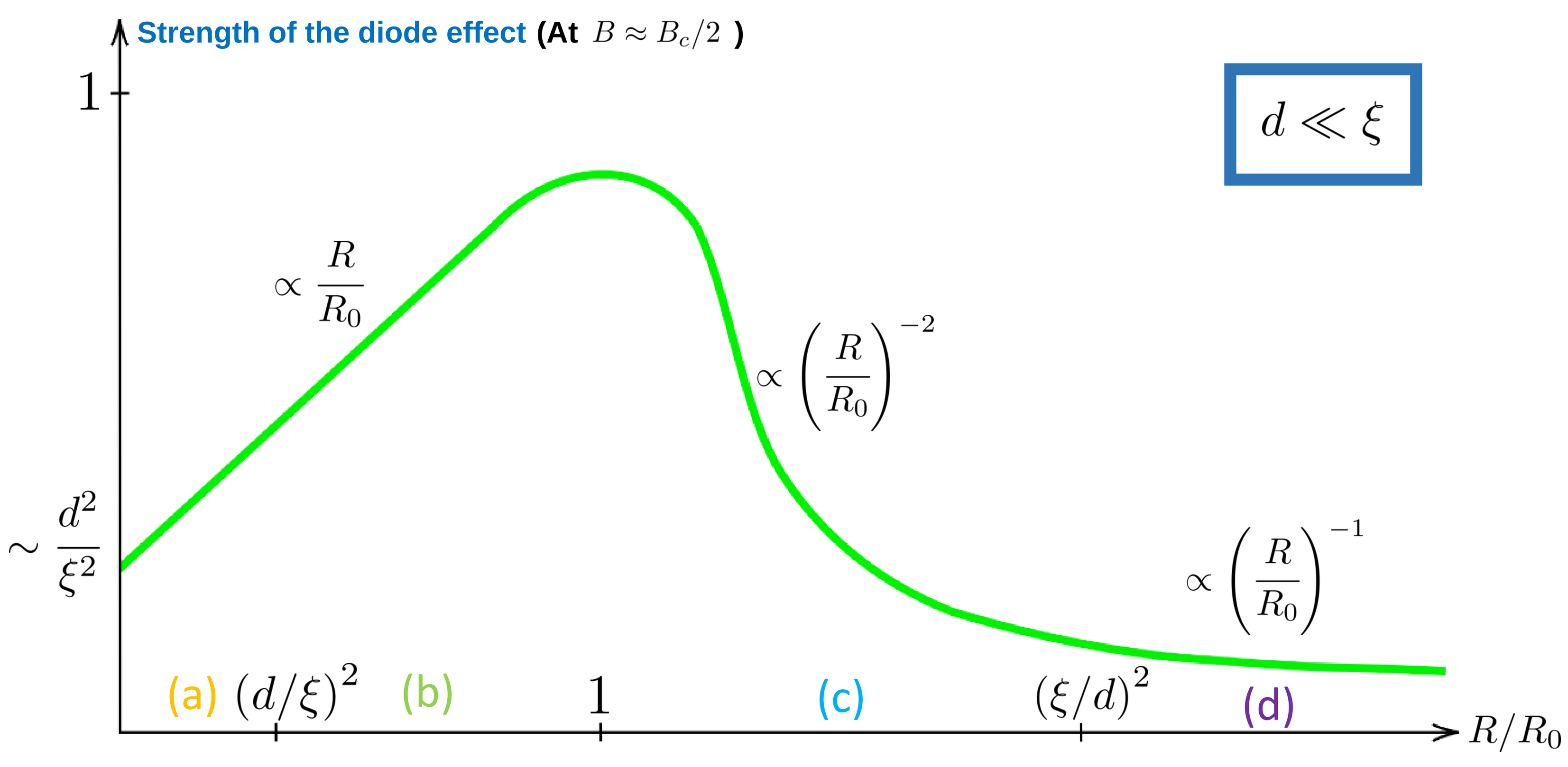

Fig. 8. Dependence of the strength of the diode effect on the transparency (resistance) of the interface in a thin bilayer.

a) $R/R_0 \ll d^2/\xi^2$ «Full transparency».
- The dominance of «inherent» inhomogeneities.
- Strength of the diode effect $\sim d^2/\xi^2$.

b) $d^2/\xi^2 \ll R/R_0 \ll 1$ «Weak nontransparency».
- The dominance of inhomogeneity, created by the interface.
- Increasing the resistance increases the inhomogeneity.
- Strength of the diode effect is monotonically increasing $\propto R/R_0$.

c) $1 \ll R/R_0 \ll \xi^2/d^2$ «Strong nontransparency».
- The dominance of inhomogeneity, created by the interface - the small contribution of superconducting electrons in the N layer.
- Increasing the resistance suppresses induced superconductivity in the N layer.
- Strength of the diode effect monotonically decreasing $\propto (R/R_0)^{-2}$.

d) $\xi^2/d^2 \ll R/R_0$ «Fully opaque».
- The dominance of «inherent» inhomogeneities. The superconductivity in the N layer is strongly suppressed, but the S layer continues to feel it.
- Strength of the diode effect monotonically decreasing $\propto (R/R_0)^{-1}$.

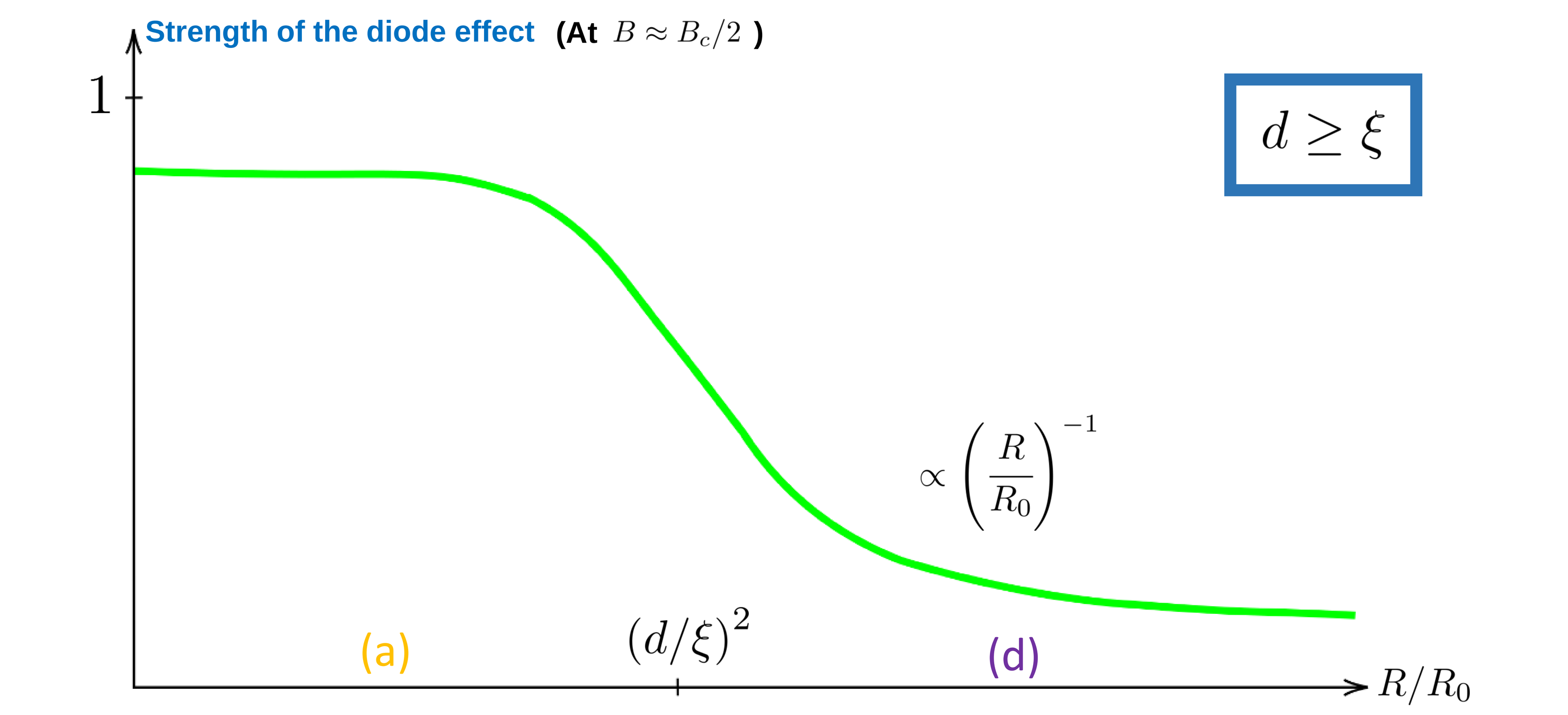

Fig. 9. Dependence of the strength of the diode effect on the transparency (resistance) of the interface in a thick bilayer.

- The area of nonmonotonicity (b) – (c) $d^2/\xi^2 \ll R/R_0 \ll \xi^2/d^2$, where the contribution of the interface was dominant, disappears with increasing thickness $d \to \xi$. The extremum disappears.
- In the transparent area (a) $R/R_0 \ll d^2/\xi^2$ «inherent» inhomogeneities dominate $\sim d^2/\xi^2$. The contribution of the interface is insignificant $\sim R/R_0 \ll d^2/\xi^2$. Strength of the diode effect does not depend on transparency.
- In «fully opaque» area (d) $\xi^2/d^2 \ll R/R_0$ «inherent» inhomogeneities in S layer dominate. Induced superconductivity in the N layer is suppressed. The situation is indistinguishable from a similar one in a thin bilayer. Strength of the diode effect monotonically decreasing $\propto (R/R_0)^{-1}$.
- The boundary resistance in the thick bilayer only weakens the diode effect. However, near the phase transition, the correlation length ξ diverges, and any bilayer behaves like a thin one.

The work was supported by the Russian Science Foundation (Grant No. 24-12-00357).

1. M. Yu. Levichev, I. Yu. Pashenkin, N. S. Gusev, D. Yu. Vodolazov «Finite momentum superconductivity in superconducting hybrids: Orbital mechanism », Phys. Rev. B 108, 094517 (2023)
2. Ya. V. Fominov and M. V. Feigel'man «Superconductive properties of thin dirty superconductor–normal-metal bilayers», Phys. Rev. B 63, 094518 (2001)